Shadow Fading Induced Capacity Reduction for an Idealized FFH/CDMA Cellular Mobile Radio System

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In this paper the effects of shadow fading on the performance of a Fast-Frequency-Hopped Code Division Multiple Access (FFH/CDMA) digital cellular mobile radio have been studied. The approach derives analytical expressions describing combined Rayleigh and shadow fading on a single narrowband channel. The effect on the performance of the wideband FFH/CDMA system is then assessed by means of multiple narrowband channel analogy. It is shown that shadow fading may cause a 50% reduction in the number of simultaneous users that can be accommodated in a cell.

KEYWORDS: Digital Cellular Mobile Radio Systems, Rayleigh and Shadow Fading, Multilevel Frequency Shift Keying, Base-to-Mobile Transmission, System Capacity Reduction

1. INTRODUCTION

In recent years a number of authors (Refs. 1-3) have considered the potential application of Fast-Frequency-Hopped Code Division Multiple Access (FFH/CDMA) as a system architecture for digital cellular mobile radio. An attractive feature of this system type, compared with other more conventional multiple accessing techniques such as narrowband frequency division multiple access, is its inherent immunity to transmission degradations caused by frequency-selective fading. However, to date, theoretical investigations of FFH/CDMA systems by individual groups have led to a wide range of predicted spectral efficiencies being reported. In this respect, there is little conclusive evidence that FFH/CDMA is superior to other architectures. More recently, a comparative study (Ref. 4) highlighting these spectral efficiency discrepancies indicated the need for further investigations of radio propagation effects to resolve some of the questions raised in reported performance predictions.

As an initial step in such an investigation, the effects of shadow fading on the transmission performance for a particular FFH/CDMA system have been considered. The present paper reports the findings which show that shadow fading may significantly reduce the number of simultaneous users that can be accommodated in a cell. The approach used to ascertain these performance predictions follows the analytical method of Goodman et al based on a multiple narrowband channel analog (Ref. 2), with an extension to include shadowing. In Section 2, equations characterising Rayleigh and shadow fading phenomena for an individual narrowband channel are used to derive propagation parameters. In Section 3, the model for synchronous base-to-mobile transmission is presented, and equations enabling performance predictions in terms of the propagation parameters are described. Using combined expressions from Sections 2 and 3, results showing the relationship between the number of simultaneous users, average signal-to-noise ratio and degree of shadowing are presented in Section 4.

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2. THE EFFECTS OF PROPAGATION CHARACTERISTICS

For microwave frequencies in an urban terrain, transmitted waves undergoing multiple reflections and refractions from obstacles form a frequency dependent standing wave pattern. Over distances of the order of tens of wavelengths, where the "local" mean signal is effectively constant, measured fluctuations in the received field strength have well-characterized Rayleigh statistics (Ref. 5). The pattern of this fading is illustrated by the rapidly varying fluctuations in Fig. 1, where successive minima are spaced approximately one-half wavelength apart, so that a moving mobile may typically encounter hundreds of these 'fast' fades in one second. In addition, these 'fast' fades have imposed on them a slower fading pattern, caused by the shadowing of large obstacles generally in the vicinity of the receiver. Experimental observations show that these frequency independent slow fades, typically tens or hundreds of metres apart, have a log-normal distribution of their "local" mean about an overall cellular "area" mean. The form of this distribution can be understood by considering a limit theorem applied to sums of randomly distributed signal power losses (Ref 6.).

For the mobile-to-base direction of transmission, proposed FFH/CDMA systems are normally assumed to be fitted with an automatic power control mechanism to compensate for shadow fading (Refs 1-2). However, for the base-to-mobile direction this

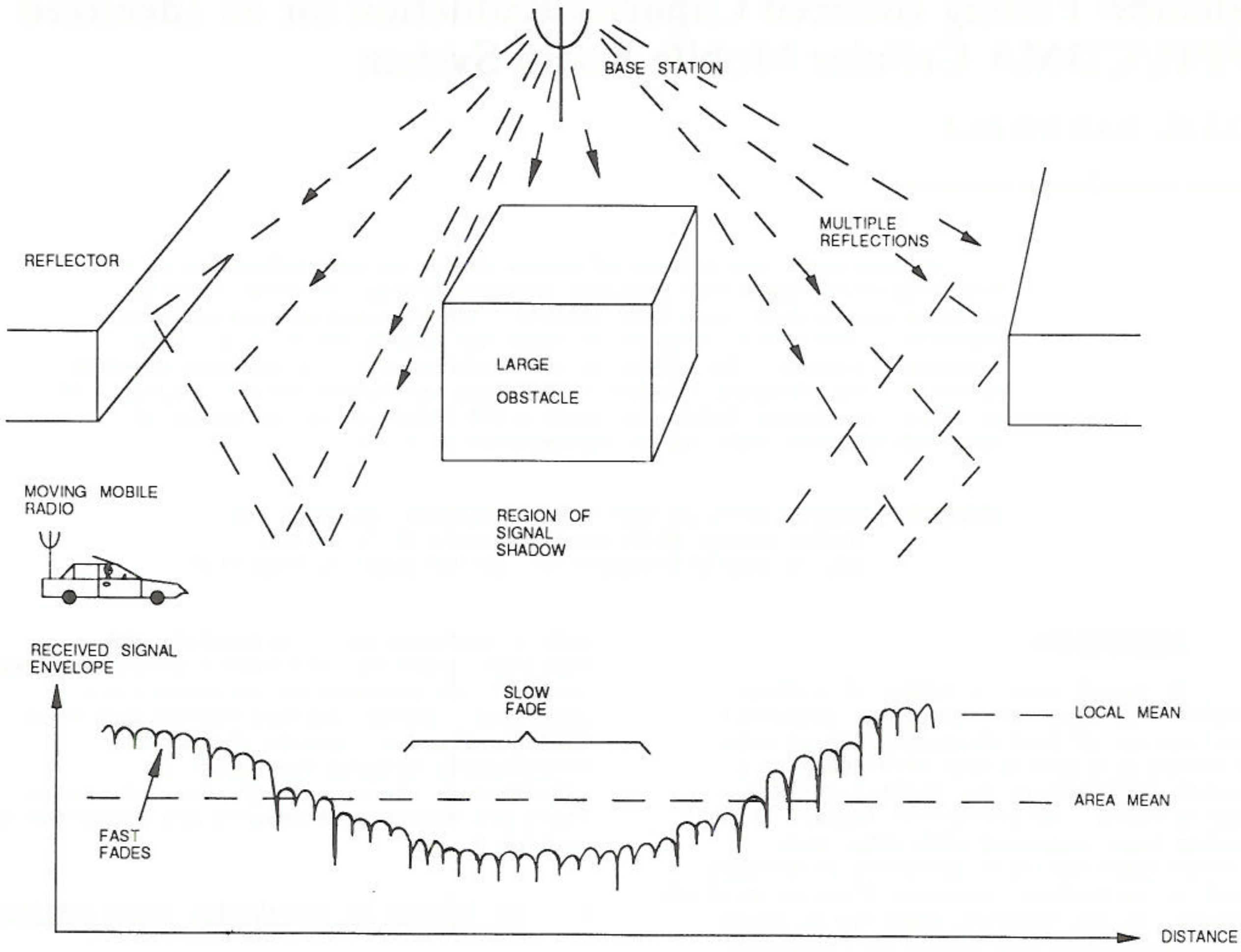

Fig. 1 - Received Signal Envelope of Mobile Passing Through a 3-Dimensional Standing Wave Pattern

assumption can no longer be applied. Therefore, in addition to the 'fast' Rayleigh fading previously considered, transmission impairments due to slow fading must be considered. These impairments are now analysed, commencing with the expression (Ref. 7) for the combined Rayleigh and log-normal probability density function p(S) of the received envelope S, of a CW transmission.

$$p(S) = \sqrt{\frac{\pi}{8\sigma^2}} \int_{-\infty}^{+\infty} \frac{S}{10^{\bar{S}_d/10}} . \exp\left\{-\frac{\pi S^2}{4.10^{\bar{S}_d/10}}\right\} . \exp\left\{-\frac{(\bar{S}_d - M_d)^2}{2\sigma^2}\right\} d\bar{S}_d \qquad (1)$$

where the local mean $\bar{S} = \langle S \rangle$ is the mean of the Rayleigh distribution

and $\bar{S_d} = 20 \log_{10} \bar{S}$.

The area mean $M_d = \langle\bar{S_d}\rangle$ is the mean of the log-normal distribution and σ is the corresponding standard deviation, typically between 6 and 12 dB (Ref. 7). It has been suggested that the extent of shadow fading, measured by σ, increases with the degree of urbanization (Ref. 8).

The Multilevel Frequency Shift Keyed (MFSK) system model considered here (described in Section 3), uses a non-coherent detector having associated ON-OFF Keying error probabilities. These are P_{em} and P_{es}, the error probabilities for a Mark and Space being sent, respectively. Assuming no intersymbol interference (Ref. 9), these probabilities may be represented as,

$$P_{em} = 1 - \int_{-\infty}^{+\infty} \frac{r}{N} . \exp\left(-\frac{r^2+S^2}{2N}\right) . I_0\left(\frac{rS}{N}\right) dr \qquad (2)$$

$$P_{es} = \int_{b}^{\infty} \frac{r}{N} . \exp\left(-\frac{r^2}{2N}\right) dr \qquad (3)$$

where N is the one-sided power spectral density of additive white Gaussian noise at the detector sampling instant;

S is the signal envelope, as previously described in (1);

r is the received envelope of signal plus noise;

b is the detector threshold level; and

I_0 is the modified Bessel function of the zeroth order and first kind.

We now define expressions for two key channel parameters, P_F and P_D, specific for non-coherent detection. From Equation (3), P_F, the probability that a signal is detected when no signal is sent (i.e. a False Alarm), is given by

$$P_F = P_{es} \qquad (4)$$

$$= \exp\left(-\frac{b_o^2}{2}\right) \qquad (5)$$

where $b_o = b/\sqrt{N}$ is the normalized detection threshold.

From Equations (1) and (2), P_D, the probability that a signal is <u>not</u> received when a signal is sent (i.e. a deletion), is

$$P_D = \int_{o}^{\infty} P_{em} . p(S)\, dS \qquad (6)$$

$$= 1 - \frac{1}{\sqrt{8\pi\sigma^2}} \int_{-\infty}^{+\infty} \exp\left\{-\frac{(\bar{S}_d - M_d)^2}{2\sigma^2}\right\} . \exp\left\{\frac{-b_o^2/2}{1 - \dfrac{1}{1 + \dfrac{2\pi N}{4} . \exp(-\bar{S}_d \ln 10/10)}}\right\} d\bar{S}_d \qquad (7)$$

using a standard tabulation of integrals, and after a transformation,

$$P_D = 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \exp(-x^2) . \exp\left\{\frac{-b_o^2/2}{1 + \dfrac{4}{2\pi N} . 10^{M_d/10} . 10^{x\sigma\sqrt{2}/10}}\right\} dx \qquad (8)$$

where all symbols have their previously defined meaning. It should be noted that P_D, as expected, has dependence on the degree of shadowing, σ. Eqn. (8) is of the same forms as Eqn. (9) in the paper by Muammar and Gupta, (Ref. 10), except the σ term has been mistakenly omitted by those authors. The equation may be evaluated numerically, or by using an approximate analytical expansion similar to Eqn. (7), (Ref. 11).

$$P_D \simeq 1 - \frac{1}{\sqrt{\pi}} \left\{ \sum_{i=1}^{3} C_i \left\{ \exp\left\{\frac{-b_o^2/2}{1 + M_d' . 10^{D_i . \sigma}}\right\} + \exp\left\{\frac{-b_o^2/2}{1 + M_d' . 10^{-D_i . \sigma}}\right\} \right\} \right\} \qquad (9)$$

where, from (Ref. 12)

$C_1 = (0.94530\ 87204\ 829)/2$
$C_2 = 0.39361\ 93231\ 522,$
$C_3 = 0.01995\ 32420\ 591,$
$D_1 = 0.00$
$D_2 = 0.95857\ 24646\ 138,$
$D_3 = 2.02018\ 28704\ 561,$

and the average signal-to-noise ratio,

$$M_d' = \frac{4}{2N\pi} . 10^{M_d/10}$$

In the case when there is no shadowing, $\sigma = 0$, and Eqn. (8) reduces to a closed form expression, analogous to Eqn. (9-5-9) in Ref. 9. That is

$$P_D = 1 - \exp\left\{\frac{-b_o^2/2}{1 + M_d'}\right\} \quad \text{for } \sigma = 0 \qquad (10)$$

For this case, by considering the equivalence of the two p.d.f.s (2) in Ref. 7 and (9-5-1) in Ref. 9, it may be readily shown that the average SNR Md', denoted by γ_o, is given by

$$\gamma_o = \langle\gamma\rangle = \frac{2\bar{S}^2}{\pi N} \text{ in dB} \qquad (11)$$

Following the procedure outlined in Ref. 2, relationships of P_D versus P_F may be constructed using b_0 as a common variable. Typical results obtained in this way from (5) and (8), for constant area SNR M_d' and shadow standard deviation σ values, are plotted in Fig. 2. Results for the same M_d' values, but with $\sigma = 0$, as determined from (5) and (10) are also shown for comparison. Note that points on the curves with $\sigma = 12$ have considerably higher P_D P_F products than those on the corresponding curves with $\sigma = 0$. In the following section these receiver curves are used to ascertain the effect of shadow fading on system performance.

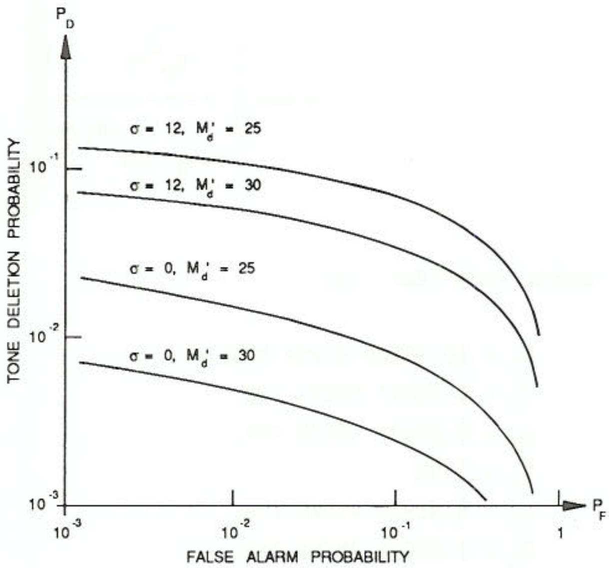

Fig. 2 - Receiver Operating Curves (σ, M_d defined in text)

3. SYSTEM ARCHITECTURE MODEL

The architecture of the MFSK FFH/CDMA system proposed by Goodman et al is shown in Fig. 3(a). During a signalling period T, input binary information, at a rate R bit/s, is buffered into a K bit "message" word. This code word is repeatedly modulo-2^K added to a user defined address sequence of L K-bit "address" words. Each of the "resultant" K bit words specifies a possible 2^K level, and is converted to a frequency tone or "chip" with duration τ. Due to the stochastic characteristic of the input information, the carrier-tones or "chips" transmitted from the multi-tone source will "hop" over the frequency-time spectrum of bandwidth W during period $T = L\tau$ seconds in a random pattern. In particular, each tone chip represents the same K-bit message word, so this pattern of redundant information achieves an 'intrinsic' frequency diversity against frequency-selective Rayleigh fades, as discussed in Section 2.

The receiver of Fig. 3(a), equivalent to a series of narrowband filters with associated detectors, converts the pattern at spread frequencies into L K-bit received words. These are modulo-2^K subtracted by the same user defined sequence of L K-bit "address" words to yield the row of L redundant K-bit "message" words originally sent. Chips from other senders are also received, however the address data, unique to each particular user, only re-spreads the hopping pattern in these cases. Considering the detection matrix, (see Fig 3(a), in the presence of other chips, the code-word chosen "is the one associated with the row with the greatest number of entries". The majority logic decision criterion chooses the correct row except when conditions, such as a significantly large number of users in the cell, causes an equal number of entries in another row, in which case an erroneous code-word can be chosen. This intracell interference, under ideal transmission considerations, sets a theoretical upper limit to the number of simultaneous users M for an average bit error ratio P_B.

However, in practice noise, Rayleigh fades and shadow fades cause transmission errors, and hence further reductions in the number of users occur. Fig. 3(b) illustrates the two fundamental types of chip error, deletions and false alarms, as described in Section 2. The occurrence of these errors affects the number of new entries and hence the majority logic decision error. By assigning respective "detection" and "false alarm" probabilities P_D and P_F, and the system parameters K bits per codeword and L chips per signalling interval, values of M users can be found for acceptable P_B, using

$$P_B \leq F(K, L, M, P_F, P_D) \tag{12}$$

and function F summarizes equations listed in Appendix A. Using suitable values for bandwidth W,

$$W = \frac{2^K}{\tau} = 20 \text{ MHz} \tag{13}$$

and input data rate R,

$$R = \frac{K}{L\tau} = 32 \text{ kbits/sec,} \tag{14}$$

then for $P_B < 10^{-3}$, optimum values of L and K (depending weakly on P_D, P_F), for a maximum number of users, were found to be L = 19, K = 8 (Ref. 2).

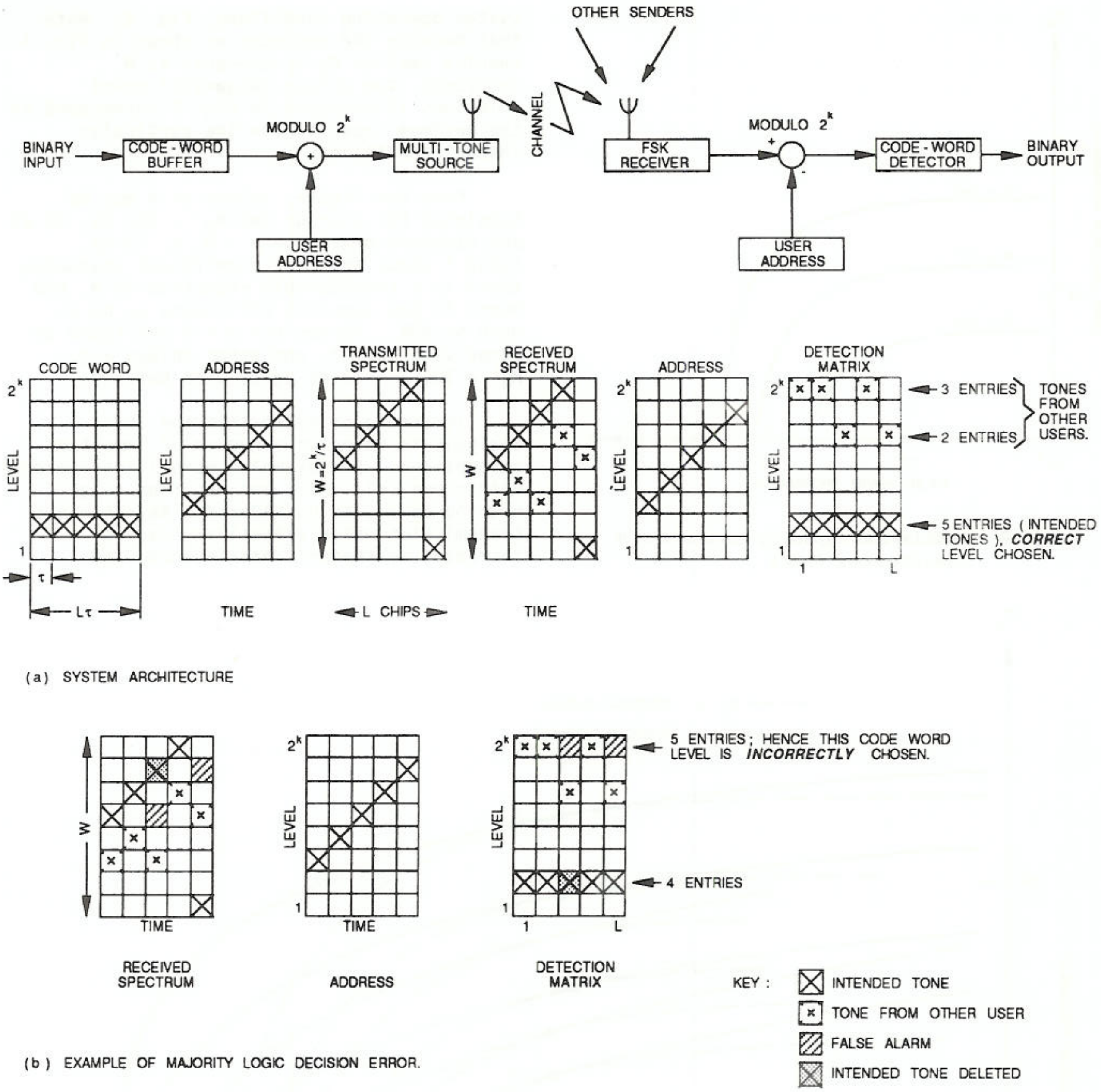

Fig. 3 - MFSK FFH/CDMA System

Using $L = 19$, $K = 8$, $P_B < 10^{-3}$ in (12), modulation capacity relationships of P_D versus P_F may be constructed (using a cubic point fitting approximation) for constant values of M. Some examples are shown in Fig. 4.

4. RESULTS and CONCLUSION

The performance criterion for this study, for a specified bit error ratio P_B, is the spectral efficiency η defined by

$$\eta \triangleq \frac{MR}{W} \text{ user bit/s per hertz per cell} \qquad (15)$$

Since, for any system, R and W are held constant, it follows that the spectral efficiency η,

$$\eta \propto M \qquad \text{users per cell} \qquad (16)$$

To ascertain values of M (number of users), for channel conditions M_d' and σ, requires the simultaneous solution of both sets of {M} and {M_d', σ} contour curves, both conveniently constructed using variables P_D and P_F. This is achieved graphically by reiteratively plotting both types of contour curves until they are tangential to each other, hence defining unique channel-

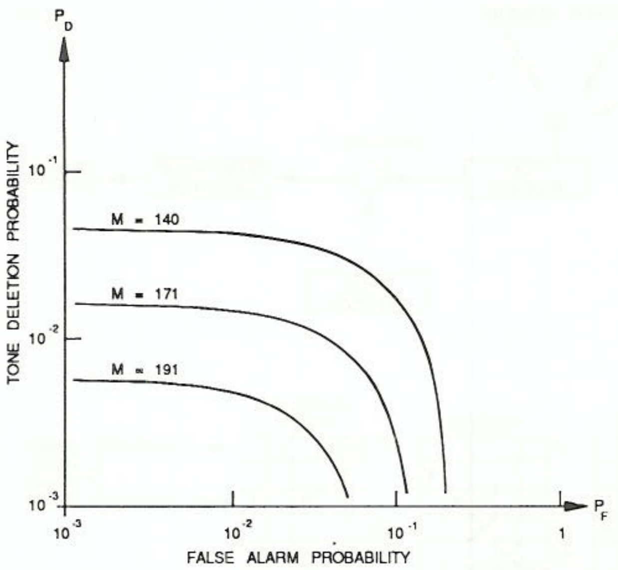

Fig. 4 - Modulation Capacity Curves for M Simultaneous Users

system operating conditions, Fig. 5. Note that because the contours as shown in Fig. 4 require smaller P_D P_F products as M increases, the unique tangential point solutions illustrated in Fig. 5 correspond to the maximum capacity for the particular channel conditions.

From the Figure, values of M may be tabulated for average SNR $M_d' = 20, 25, 30$ dB and standard deviation $\sigma = 0, 6, 12$ dB. Table 1 shows that for significant shadowing there is a considerable reduction in M, and hence in the spectral efficiency η, by as much as 50%. Curves for $\sigma = 0$ are found to agree with Ref. 2, and other values e.g.: $M_d' = 30$, $\sigma = 12$ dB agree with Ref. 10.

It should be noted that the above analysis is based on a number of important assumptions. For example, mutual interference within one cell only, and not from neighbouring cells, is considered. Asynchronous transmission which effectively causes increased co-channel interference; and

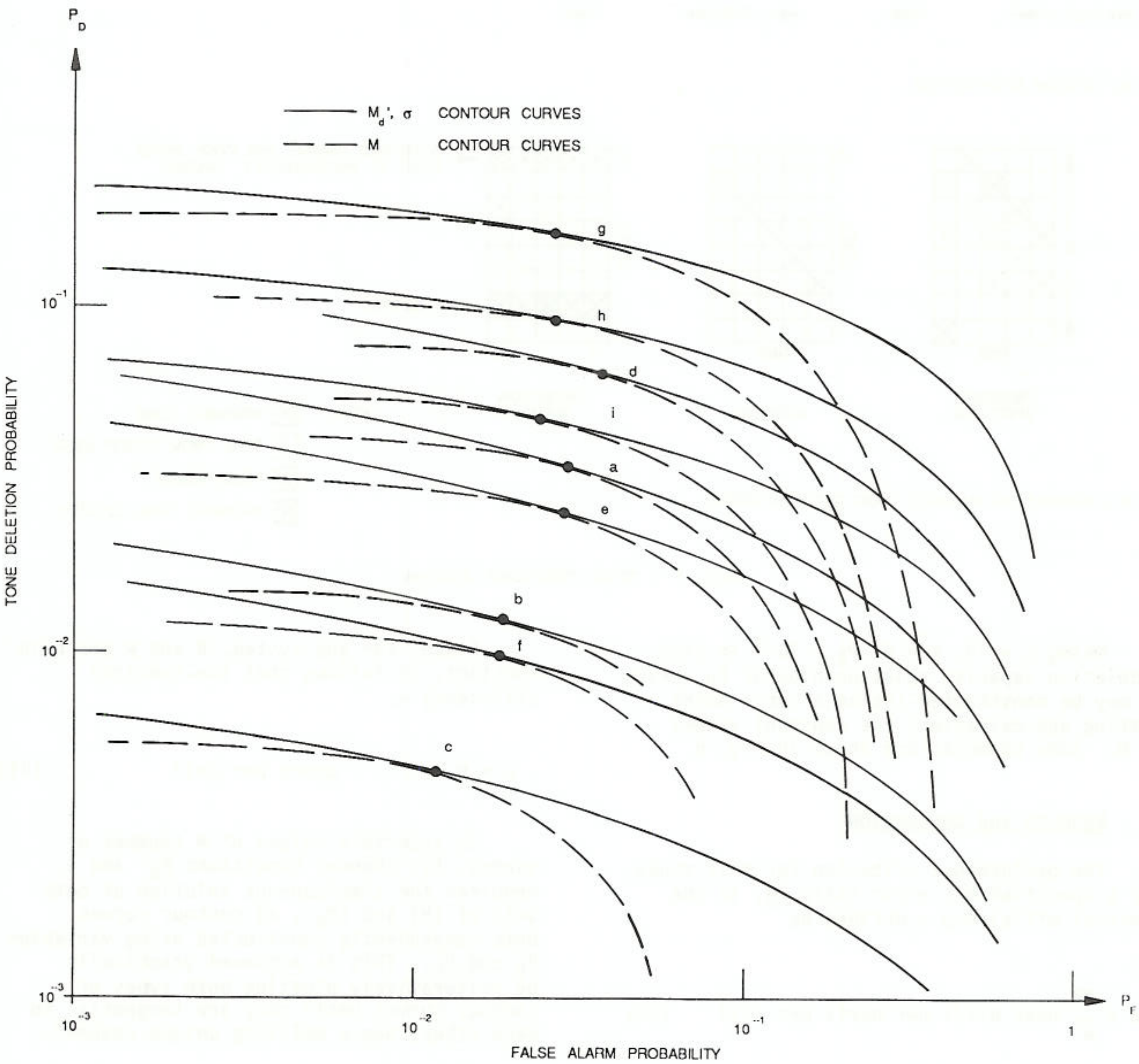

Fig. 5 - Superposition of Receiver Operating (M_d, σ) and M Modulation Capacity Curves

TABLE 1 - Channel-System Operating Conditions : Values of M and M_d' and σ, from Fig. 5.

M	$\sigma = 0$	$\sigma = 6$	$\sigma = 12$
$M_d' = 20$	(a) 140	(d) 113	(g) 70
$M_d' = 25$	(b) 171	(e) 149	(h) 100
$M_d' = 30$	(c) 191	(f) 176	(i) 130

adjacent channel interference, particularly severe for the "near-far" situation, are not considered. The details of these assumptions may be found in Refs. 2,3 and 13. Other system considerations including coherence bandwidth, Doppler shift, delay spread and the effects of coding are discussed in Refs. 14-17. In regard to the present results, because of these simplifying assumptions, the relative effect of the shadow fading is of greater significance than the absolute single cell M or η results. It is considered that this relative effect would remain essentially valid for the more detailed system models.

APPENDIX A

Error Rate Formulae from Goodman et al (Ref. 2)

Probability of insertion due to interference:

$$p = [1 - (1 - 2^{-K})^{M-1}](1 - p_D)$$

Probability of insertion due to interference or false alarm:

$$p_I = p + p_F - pp_F.$$

Probability of m entries in a spurious row:

$$Ps(m) = \binom{L}{m} p_I^{\,m} (1 - p_I)^{L-m}$$

Probability that no unwanted row has as many as n entries:

$$P(n,0) = \{\sum_{m=0}^{n-1} Ps(m)\}^{2^K-1} \quad ; \quad n > 0.$$

Probability that n is the maximum number of entries in an unwanted row and only one unwanted row has n entries:

$$P(n,1) = (2^K-1)Ps(n)\,\{\sum_{m=0}^{n-1} Ps(m)\}^{2^K-2} \quad ;$$

$$n = 1,2,\ldots,L.$$

Probability of i entries in the correct row:

$$P_c(i) = \binom{L}{i} (1 - p_D)^i \cdot p_D^{\,L-i}$$

Upper bound on bit-error rate:

$$P_B \leq \frac{2^{K-1}}{2^K - 1} \left\{1 - \sum_{i=1}^{L} P_c(i)[P(i,0) + \frac{1}{2} P(i,1)]\right\}.$$

A summary of the above formulae, which assume synchronous transmission and no intercell interference, is thus

$$P_B < F\,(K,\ L,\ M,\ P_F,\ P_D)$$

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<u>BIOGRAPHY</u>

DAVID BAKEWELL was educated at the University of Melbourne where he was awarded the degree of Bachelor of Engineering (Electrical) with Honors in 1983. After graduation he joined Telecom Australia and in 1985 commenced work at the Research Laboratories, Radio Systems Section. Currently, he is an Engineer Class 2 in the Optical Systems Section, and is concerned with the application of laser amplifiers to the Australian optical fibre network.